

Complex Action Support from Coincidences of Couplings

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Abstract

Our model [1] [2] with a complex action in a functional integral formulation with path integrals extending over all times, both past and future, is reviewed. Several numerical relations between coupling constants are presented as supporting evidence. The new evidence is that several more hitherto unexplained coincidences are explained by our model:

1) The “scale problem” is solved because the Higgs field expectation value is predicted to be very small compared to say some fundamental scale, that might be the Planck scale.

2) The Higgs VEV need not be just zero, but rather is predicted to be so that the running top-quark Yukawa coupling just is about to e unity at this scale; in this way the (weak) scale easily becomes “exponentially small. Instead of the top-Yukawa we should rather say the highest flavour Yukawa coupling here.

These predictions are only achieved by allowing the principle of minimization of the imaginary part of the action $S_I(history)$ to to a certain extent adjust some coupling constants in addition to the initial conditions.

If supersymmetric partners are not found at LHC it would strengthen the need for a “solution” of the hierarchy problem in our direction of an explanation via a finetuning scheme inside the Standard Model, from say minizing “the imaginary part of the action” in our complex action model.

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1 Introduction

Masao Ninomiya and myself [1] [2] have since some time worked on the idea that the action going into the Wentzel-Dirac-Feynman-path-integral [3][4][5] should be *complex*, rather than as normally assumed, real. Alone to argue that such a model would not lead to so drastic predictions deviating from usual observations, that it is not falsified immediately, needs a highly nontrivial speculative argumentation. One step is to first see that the effect of the imaginary part of the action first of all is *determine the initial conditions* by selecting which solution to the equations of motion is to be realized, and that according to a minimization rule of this imaginary part of the action $S_I(\text{history})$, a quantity depending a priori on what goes on at all times. Then, however, we can suggest that the time close to the effective Big Bang time would be much more important for determining the development, so that not much freedom in “choosing” it would be left to cause effects of the imaginary part to be seen at later times, is not unnatural in our model. To have a complex action would in fact be simpler in the sense that as far as the integrand $\exp(\frac{i}{\hbar}S(\text{path}))$ of the path way integral is anyway complex, whether the action $S(\text{path})$ is allowed to be complex or only to be real. Then one could namely take the philosophy, that the most “fundamental” object were the *integrand* rather than the action and would even in that philosophy not have to impose any strange extra assumption on this integrand such as the usual one that it be a pure phase factor. These type of ideas leading to influence coming from the future too were indeed forerun [6][7] by works by Don Bennett and myself and by bayuniverse theory [8][9][10] [11] [12].

It is the main purpose of the present article to put forward a couple of numerical coincidences that support the complex action model in the sense that they can be derived from the latter under mild and natural assumptions within it and which seem to agree with the experimentally measured parameters.

One such relation I already published in [13] and that means that there is relation between the light quark masses and the electron and nucleon masses in a way that would be hard to understand as more than accidental in any other model. Also in that article were mentioned the funny accident that the “knee” [21] in the cosmic ray spectrum [22] - an energy at which the intensity of cosmic ray impacts seeming starts falling faster with energy than below - happens to be very close in order of magnitude to the effective threshold for significant Higgs boson production.

The main new point of the present article is, however, what we could consider a solution of the scale problem - related to the hierarchy problem of why the weak energy scale (or say Higgs mass scale) is so enormously low compared to say the Plack scale, or if there were some GUT then compared to the GUT-scale.

The main point really is that we argue in the development of the complex action model for that the *square* of the Higgs boson field averaged is the quantity that should be minimized so as to fix say the bare Higgs mass square and thereby the Higgs field expectation value $\langle \phi_h(x) \rangle$ to minimize the square. It is intuitively obvious that minimizing the *square* leads easily to make the average of the Higgs field itself become small. We shall see, however, that nearer estimates involving the influence of virtual top-quarks in the vacuum very likely leads to a correction that potentially could make it non-profitable in searching for the lowest possible average of the *square* of the Higgs

field $\langle \phi_h(x)^2 \rangle$ to just decrease the expectation value of the Higgs field itself unlimited towards zero. Rather, when the top-Yukawa coupling gets sufficiently large, it could and we shall suggest that it does go oppositely, and it thus pays to stop decreasing the Higgs field expectation value $\langle \phi_h(x) \rangle$ at the situation wherein the running top-Yukawa coupling at the scale of this $\langle \phi_h(x) \rangle$ passes a certain value of order unity.

Since the running of say the top Yukawa coupling $g_t(\mu)$ is “logarithmically slow” the value of an energy scale determined by putting such a running coupling to a special value, such as one of order unity, can easily lead to an “exponentially small” energy scale. It is in this way our work points to a possible explanation for how the weak scale related to say the Higgs mass or the Higgs field expectation value could become naturally “exponentially small”. That is to say our model based on the complex action leads to a solution of what we may call the “scale problem” of “Why is the Higgs or weak scale so extremely low compared to Planck scale?”. Having such an explanation of an *adjustment mechanism* making the Higgs mass say small one should of course in principle use it order by order in perturbation theory in the Standard Model. Thus although there would still be a problem of tuning in the bare Higgs mass square by huge terms compared to the renormalized squared mass, we would in our model have an explanation for why we have to renormalize in this “fine tuning” way and thus in a certain philosophical sense we would have “solved the hierarchy problem”, in spite of there still being huge renormalizations (quadratically divergent contributions in the bare mass square).

Since we shall claim to explain the smallness compared to the Planck scale of the Higgs field vacuum expectation value - as what minimizes the Higgs field squared and averaged-, we can claim that we propose a picture for which there would be a stronger call if the susy-partners are not found at LHC. Then namely we cannot even solve the hierarchy problem by susy, and it will be really needing with some finetuning law rather than avoiding finetuning totally. We have got the minimization of the S_I to lead to finetuning, that may thus help solving even hierarchyproblem at the end.

After in section 2 and section 3 to have reviewed very shortly the complex action model by Ninomya and myself [1] [2] with special emphasise in section 3 on the approximation that the Higgs mass term is supposedly dominant, we shall in section 4 introduce the assumption that also some coupling constants shall be adjusted to minimize the imaginary part of the action S_I . Next in section 5 we discuss how we expect the average of the square of the Higgs field to vary as a function of the average of the Higgs field itself, when we vary say the (real part of the) bare Higgs mass square $m_h^2|_R$, i.e. the coefficient of the Higgs field square term in the Lagrangian density of the Standard Model. It is in this section 5 we achieve the main result of predicting the Higgs expectation value from minimizing the average of the square of the Higgs field and predict the scale to be the one where the top-Yukawa coupling is of order unity. Then in section 6 we shall discuss whether a model with a fixing of initial conditions as well as coupling constants to some extent like our model will not lead to some strange effect, such as cleaning the universe for neutrons or oppositely making all the proton and electrons into neutrons. This worry turns out to be well worth worrying about. The point, however, turns out to be that there is just such a relation between certain coupling constants - the light quark masses the electron mass and the energy of the valence quarks in the nucleon - so that it does not matter if we let the neutron decay or not for evaluating the imaginary part of the action S_I . This

formula is reviewed from reference [13] in section 7. Before concluding we put forward a couple of other adjustments to make our minimizing Higgs field square integrated principle consistent in section 8. Especially it is new that I here in subsection 8.1 seek to show that our model only shall be consistent provided the binding of nucleons into nuclei is remarkably weak, as we would say it is. Finally in section 9 we shall conclude that there has actually collected itself a few numerical coincidences strongly suggesting that there ought to be something about the complex action model, since otherwise why should they be satisfied even approximately?

2 Short review of complex action model

Looking at the Wentzel Dirac Feynman path integral [3, 4, 5]

$$\int \exp\left(\frac{i}{\hbar} * (S_R[path] + iS_I[path])\right) \mathcal{D}path \quad (1)$$

for the case that the action is considered being complex

$$S[path] = S_R[path] + iS_I[path] \quad (2)$$

(where of course S_R and S_I just are the real and imaginary parts of the action S) it is clear that almost whatever the interpretation procedure for the use of this path-integral may be taken to be it will be so that only the paths for which the imaginary part $S_I[path]$ is close to being minimal - at least among those paths, which corresponds to some classical solution - can contribute significantly. In the light of that having the \hbar in the denominator suggests the coefficient $\frac{i}{\hbar}$ to be in some “human terms” very large we would even expect that probably only the very minimum value of the imaginary part of the action S_I will be realized for any path of significance. That fact we take to mean physically that the history of the universe which is actually realized - the one we live through - thinking in a classical approximation must be that history among the ones agreeing with the equations of motion at all that has the minimal value for the imaginary part $S_I[history]$. This is a first remarkable result of the investigation of the complex action model, that it becomes *a theory of the initial conditions* and not only for the equations of motion as usual theories have it.

Next remark that since in the way we did it with inclusion of both past, present and future into the time-integral defining the action (2)

$$S[path] = S_R[path] + iS_I[path] = \int_{-\infty}^{\infty} L(t)dt \quad (3)$$

where of course now $L(t) = L_R + iL_I(t)$ is the complex Lagrangian, the selection of the *initial conditions and thereby solution being selected to be the realized one by having minimal S_I now depends on the future too..* This indeed means as a very remarkable property of theories of this type that it will be as if there is some arrangement going on in the world achieving certain happenings in the future. That is to say that our complex action model has in it *backward causation*. If for instance as we earlier suggested Higgs particle production in big amounts (in [13] the number of order 300000 Higgses is mentioned as

possible critical number in the sense that more Higgses in a single machine should presumably be avoided, if it is at all possible by adjustment of the initial conditions) should be avoided by such prearrangement of the historical development of the Universe. There should essentially appear a bit of miraculously bad lucks for Higgs producing machines (making more than the critical number order of magnitude), so that they some way or another by bad luck turn out not to come to function.

There has been in the development of the understanding of the consequences of our complex action model some difficulty in speculating out how to avoid that these effects of backward causation or prearrangements, avoiding e.g. Higgs production, become so big that there would be no chance that they should not have been observed. If indeed our complex action model predicted that there would be several happenings that would be avoided while other types of events would/should show up more copiously than we would expect from statistics our model would be killed already from the very fact that we do not see much of such effects in daily life. If we should make an a priori guess as to the size of the imaginary part S_I of the action it would be that the real part S_R and the imaginary part S_I would have coefficients to the various terms in the Lagrangian density being of the same order of magnitude. We hope that we can still uphold such a crude hypothesis of the coefficients in the Lagrangian density, which would mean that the coefficients (coupling constants, mass squares) would just be complex numbers with phases of order unity measured in radian. But such a situation would in very naive thinking lead to the a priori exponential suppression factors of the integrand in the path way integral, and that with a very big exponent, which even has the expected tremendously big factor $1/\hbar$ in front. Really it would mean that the exponential suppression would be by the exponent of a number of the order of the phase variation of the waves in quantum mechanics for the system in question. That would be enormously much and the observation of the effects of backward causation could hardly be avoided in daily life. We hope to have developed at least some - may be a bit speculative - arguments that it is not excluded that there are very strong suppressions of the backward causation effects relative to this just mentioned naive estimation. The main argument for strong suppression of the effects of backward causation from the principle of minimizing $S_I[history]$ is that supposing that we have to have the equations of motion fulfilled what happens at one moment of time and at another moment of time gets related simply by integrating the equations of motion. If therefore something shall be arranged at one moment of time it leaves less freedom of adjustment of the initial conditions for the instalment of special happenings at another moment of time. Since the Universe already has existed for a very long time compared to the periode of time about which we have sufficient knowledge to be able to discover effects of backward causation or prearrangements the strength of the backward causation of which we can hope to be aware gets already by this consideration of many competing eras strongly suppressed. In order to have agreement with the fact of increasing entropy we have to speculate that the eras of most importance for deciding the history with the minimal S_I , i.e. the realized history, were the ones just after a possible big bang or more precisely the eras close to the very special and hot conditions etc around the time of which we usually think as the big bang time. This hypothesis that the eras near the special or big bang time dominate the selection of the history is not at all unlikely inside our complex action model, because at that time fields such as inflaton field or plasmas

had very high energy densities compared to the present time. That means then that a Lagrangian density having the same dimension as the energy density would very likely get enormously much bigger contributions during such a hot era than under the later cold eras even if there would be a compensation due to the Universe being bigger and the time spans large later on. In fact the energy density and thus presumably also the Lagrangian density - real as well as imaginary parts - goes in say radiation dominated universe as

$$\mathcal{L}(x) \propto 1/a^4 \quad (4)$$

where a is the size/radius of the universe. If we thus take for simplicity that the time span relevant is of order a like the spatial size, the integral to evaluate the action - real or imaginary part - becomes of the crude form

$$\int \frac{1}{a^4} d^4x \approx \int \frac{da}{a}. \quad (5)$$

This means that it gets about similar order of magnitude contributions from each order of magnitude of a or in our crude thinking here from each order magnitude in the time span. But if it is so there is a logarithmic divergence towards the big bang. This was very crude but it means that there could exceedingly easily be some of the time intervals from us to see very close to the big bang which could dominate.

3 The special importance of the Higgs field squared

In our times when it is the fields of the Standard Model that seemingly plays the most important role a concrete guessing for the action is of course the Standard Model action

$$\int \mathcal{L}(x) d^4x \quad (6)$$

where the Lagrangian density $\mathcal{L}(x)$ has several terms of which we just mention a few to illustrate

$$\mathcal{L}(x) = -\frac{1}{4g_s^2} F_{\mu\nu}^a(x) F^{a\mu\nu}(x) + \dots + |D\phi_H|^2 - m_h^2 |\phi_H|^2 + \dots \quad (7)$$

It would therefore be the most natural suggestion that in our times the imaginary part of the action S_I simply gets its main contributions from terms of precisely the same form just that it is now the imaginary part of the coefficients such as $\frac{1}{4g_s^2}$ and m_h^2 i.e. $\frac{1}{4g_s^2}|_I$ and $m_h^2|_I$ which gives us the imaginary part of the action

$$S_I[path] = \int \mathcal{L}|_I(x) d^4x \quad (8)$$

where now

$$\mathcal{L}(x) = -\frac{1}{4g_s^2}|_I F_{\mu\nu}^a(x) F^{a\mu\nu}(x) + \dots + Z_I |D\phi_H|^2 - m_h^2|_I |\phi_H|^2 + \dots \quad (9)$$

(On the kinetic term, which is often normalized to have unit coefficient, we had of course to insert for the case of the imaginary part a parameter or coefficient Z_I , since we cannot normalize the field - in the example the Higgs field $\phi_H(x)$ to the imaginary part, also.).

As already suggested in foregoing section it would be natural to expect the real and imaginary parts of the coupling coefficients to be of similar order of magnitude. There is, however, one case in which this may not be trustable, if we thought of the renormalized coefficients, and that is the case of the Higgs mass term. The renormalized Higgs mass is namely surprizingly small in the sense of the scale problem being the problem of why it is so very small compared to, say, the Planck mass. Remember it is a kind of mystery that it is so small, a mystery needing one explanation or an other (we shall actually give an explanation attempt ourselves in the next section) and thus it is far from obvious whether we should believe that the same mysterious mechanism making the renormalized real part of the mass square of the Higgs small should also work for the imaginary part. But most importantly we should guess that it is the bare Higgs mass coefficient that has the about equally big real and imaginary part rather than for the renormalized or dressed mass square. Now truly the hierarchy problem is about that we can not get the bare mass square and the renormalized mass square “small” - meaning of the order of the weak scale - simultaneously unless we have invented some way (such as susy) to make the renormalization correction “small”. But it is only the renormalized real part that we know is “small” so unless we have the very clever mechanism making the renormalization correction “small” we expect that the imaginary part of the mass square is not “small” at all. The most suggestive expectation is indeed that the imaginary part of the mass square of the Higgs is say Planck scale big, renormalized or not. But that then makes the term $\dots + m_H^2|_I|\phi(x)|^2 + \dots$ in the imaginary part of the Lagrangian density become enormous from the point of view of the 100GeV scale say. This not believing in any “mysterious” mechanism making the imaginary part $m_H^2|_I$ small means that it becomes about 10^{34} times bigger than the other terms for energies of the TeV or so.

This means that under the experimental conditions of daily life and realistic accelerators and even essentially all astrophysics of today presumably not even excepting cosmic rays we must count that the Higgs mass (square) term in the Lagrangian density completely dominates w.r.t the imaginary part. We can therefore except only for perhaps times deep inside the first second in cosmology use the approximation for the imaginary part of the action

$$S_I[path] = -m_h^2|_I \int |\phi_H(x)|^2 d^4x. \quad (10)$$

Since the universe has not been filled with huge amounts of Higgses we shall take it that the sign of the coefficient $m_h^2|_I$ is so that the term gets positive for big Higgs field, so that the minimization of S_I resulting from our model would tend to make there be only few Higgses. That is to say that in the sketched notation (10)

$$m_h^2|_I < 0. \quad (11)$$

Now most of the Universe is today practically vacuum or at least very close to being just empty space. In the vacuum, however, there is a fluctuating Higgs field $\phi_H(x)$ - that does as we know not even fluctuate around zero, but rather around a “small” value $246\text{ GeV}/\sqrt{2}$ in the sense of smallness just described (i.e. compared to Planck units). Because of the enormous spacewise dominance of the vacuum today we expect the most important term in the imaginary action to be minimized to be the approximation:

$$S_I[path] \approx -m_h^2|_I < |\phi_H(x)|^2 > \int d^4x \approx -m_h^2|_I < |\phi_H(x)|^2 > VT \quad (12)$$

where VT is the four volume of space-time (of course V is the volume and T the time of duration of the world.).

4 Let also the S_I -minimization Influence the Couplings

In principle the expectation value $\langle |\phi_H(x)|^2 \rangle$ in vacuum is given alone by the coupling constants of the theory, except for the case that there are several vacua, and thus it looks at first that there is no way to choose the *initial state conditions* so as to arrange anything for say this average of the square of the Higgs field $\langle |\phi_H(x)|^2 \rangle$ to be small/minimal. There are though many ways in which such a dependence on the initial state conditiond for even the vacuum can “sneak in” by some effective coupling constants depending on initial conditions:

It happens in baby universe theory, or if there were indeed very many possible vacua like in the landscape story by Carroll, Smolin, Susskind [14], or if just the fundamental physics were somehow say a pregeometry with many more degrees of freedom than the ones we know so far. There could easily be some degrees of freedom, that could somehow set the vacuum to have various properties(such as say a small Higgs field square average).In string theories there is a lot of plays with “moduli”, which are degrees of freedom that function as coupling constants. Such moduli would clearly - being considered as part of initial conditions and thus adjustable in our model to minimize S_I - be able to adjust the Higgs field expectation value.. By nature adjusting some such moduli according to the minization of S_I it would namely also adjust the Higgs field expectation value, say, since that would be a function of the moduli. So in such a way the effects of adjusting to minimize L_I or S_I could easiliy “sneak in” via some sort of moduli.

If we subscribe to one of these kinds of possibilities for the vacuum having some yet to be understood degrees of freedom behind it, it would mean that we should for the purpose of minimizing S_I consider at least some of coupling the constants or better some combinations of them or parameters parametrizing them as (dependend on) initial conditions. That should then simply mean that at least still keeping some relations between the couplings perhaps there would be some freedom in varying the couplings, that then in our model should be adjusted by minimizing the imaginary part of the action in the same way and together with the genuine initial conditions.

That is to say that we could for instance imagine that the *real part of the Higgs mass square* $m_h^2|_R$ might be adjustable and should be determined so as to minimize S_I . That would in our approximation (10) mean that the real part of the Higgs mass square should be adjusted so as to minimize the average of the square of the Higgs field $\langle |\phi_H(x)|^2 \rangle$.

So our model predicts that for instance the bare Higgs mass square coefficient gets tuned in to make the avegrage of the squared Higgs field $\langle |\phi_H(x)|^2 \rangle$ minimal. It does not really matter so much whether it is just the Higgs mass or something related, but at first we have intuitively that it should be very favorable for making $\langle |\phi_H(x)|^2 \rangle$ small to make the Higs field vacuum expectation value $\langle \phi_H(x) \rangle$ (without squaring) small or zero. So we could also for instance think of the expectation value as the quantity that gets adjusted, since it should not matter much what we think of as what we can screw on.

5 The Squared Higgs field expectation value as function of the Higgs field VEV

5.1 Crudest approximation

How do we now expect the square of the Higgs field averaged in vacuum $\langle |\phi_H(x)|^2 \rangle$ to depend on the average of the Higgs field itself $\langle \phi_H(x) \rangle$ without the square in the range of rather small values in which we are most interested ? The first estimate is simply to ignore quantum fluctuations and say that of course we simply have ignoring fluctuations

$$\langle |\phi_H(x)|^2 \rangle \approx |\langle \phi_H(x) \rangle|^2 (\text{ignoring fluctuations}). \quad (13)$$

If this approximation were true, one would predict from our model of course, that the expectation value of the Higgs field in vacuum $\langle \phi_H(x) \rangle$ should be zero. Note that in the light of thinking on the Planck units as the fundamental units in nature this prediction is indeed already a very good prediction answering to the main problem of why the Higgs expectation value is so enormously small compared to the Planck scale expectation. However we know from the whole application and reason for our belief in the existence of the Higgs field at all that the Higgs field vacuum expectation value shall not be zero but just very small compared to the Planck unit. Before going to a more accurate calculation of the Higgs field vacuum expectation value (VEV), let us shortly remember that really we should rather than the VEV $\langle \phi_h(x) \rangle$ predict some parameter in the Lagrangian density, which really here means the real part of the mass square of the Higgs $m_h^2|_R$. Now you can obtain in the classical approximation (13) the zero VEV for all positive $m_h^2|_R$. It would first be in some further degree of accuracy that we would get the value the real mass square $m_h^2|_R$ giving the minimal $\langle \phi_h(x)^2 \rangle$. Since classically this average of the square $\langle \phi_h(x)^2 \rangle$ goes as a constant for $m_h^2|_R > 0$, i.e. non-tachyonic case, any very small S_I contribution could very easily drive it to one end or the other of the interval giving the minimum. Thus it would have a high probability that for instance the mass square $m_h^2|_R$ would be driven to just zero. In this sense already in the classical approximation (13) we have argued not a finetuning mystery, but rather a likely consequence in our model to obtain

$$\text{Classically :} \quad m_h^2|_R \approx 0 \quad (14)$$

5.2 Including fluctuations

Now my claim is that we have to make a bit more accurate estimate of the Higgs field squared expectation value $\langle |\phi_H(x)|^2 \rangle$ as a function of the vacuum expectation value $\langle \phi_H(x) \rangle$ without the square. There may be several corrections since the fluctuation of the Higgs field

$$\text{"fluctuation"} = \langle |\phi_H(x)|^2 \rangle - \langle \phi_H(x) \rangle^2 = \langle |\phi_H(x) - \langle \phi_H(x) \rangle|^2 \rangle \quad (15)$$

does not have to be independent of the parameters such as $m_h^2|_R$ which is used to let the expectation value $\langle \phi_H(x) \rangle$ vary. Realistically the fluctuations will actually be huge

compared to the classical contribution $|\langle \phi_H(x) \rangle|^2$ to the average square $\langle |\phi_H(x)|^2 \rangle$ in the situation known phenomenologically of a surprisingly small Higgs field average. The size of the fluctuation will then mainly be given by the fourth order term $-\frac{\lambda}{8}|\phi_H(x)|^4$ in the Lagrangian density and thus be approximately independent of the mass- coefficient $m_h^2|_R$. Still the mass term has some effect, but there is yet another type of effect which could be even more important in the region of very small VEV $\langle \phi_H(x) \rangle$ and this is the most interesting. That is the higher order effect of the the fluctuation contribution due to virtual top-quarks say in the vacuum. Certainly there will be in the vacuum some top-quark anti-top-quark pairs virtually. It may be better to think of it by saying that the top-quarks in the Dirac sea are in quantum superpositions of having different positions and so there are quantum fluctuations in these positions at least. That means that there are top quarks here and there in a quantum random way. Around these top-quarks there will then be their Yukawa-Higgs fields meaning that in the neighborhood of the in this way virtual top-quarks the normal Higgs field value will be a bit suppressed. Of course then compared to the average there will to cope with the definition of the average be a bit enhanced Higgs field in the regions where there is accidentally relatively few virtual tops. Obviously this is expected to mean that the stronger the coupling of the top to the Higgs, i.e. the stronger the top-Yukawa-coupling g_t relevant for the scale considered, the more fluctuations there will be due to the virtual top-quark Yukawa fields. If the top-Yukawa coupling is truly so small that the effect of it is only a small correction it may not matter much for how the expectation of the square $\langle |\phi_H(x)|^2 \rangle$ behaves as function of the average $\langle \phi_H(x) \rangle$; but if the Yukawa coupling becomes of the order of unity in the sense that the virtual top fluctuation contribution becomes of the same order as the contribution $|\langle \phi_H(x) \rangle|^2$ via the classical approximation to the average of the square $\langle |\phi_H(x)|^2 \rangle$, this virtual top contribution could be significant. Actually one could imagine that once the running top-Yukawa $g_t(\mu)$ has become of order unity, then contrary to classical expectation the average of the square $\langle |\phi_H(x)|^2 \rangle$ would no longer be an increasing function of the expectation value $\langle \phi_H(x) \rangle$ but rather decreasing instead. The point is that varying the expectation value $\langle \phi_H(x) \rangle$ the top quark mass increases with increasing expectation value and thus the density of virtual top quarks decrease, leading in turn to a decrease of the fluctuation in the Higgs field due to these virtual top quarks. Thus if this effect of the virtual top quarks is sufficiently strong it might overcompensate for the classical effect w.r.t how the average square $\langle |\phi_H(x)|^2 \rangle$ varies as function of the average $\langle \phi_H(x) \rangle$. When these two compensating effects just balance there will be a minimum or a maximum in the average of the square $\langle |\phi_H(x)|^2 \rangle$ as a function of the average itself $\langle \phi_H(x) \rangle$ of the Higgs field. Whether it becomes a maximum or a minimum depends on the sign of the β -function for the top-Yukawa coupling at the relevant scale μ . We shall indeed easily see by using the expression for the top-Yukawa beta function that in the weak scale range, which is of interest for us, it would if the coupling gets sufficiently strong to make either a minimum or a maximum.

We shall see that it indeed becomes a minimum: We shall use the perturbative result [20]

$$\beta_t^{(1)} = -\left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right)g_t + \frac{9}{2}g_t^3 + \frac{3}{2}g_t g_b^2 \quad (16)$$

with SU(5)adjusted convention for the coupling constants, where we have defined

$$\frac{dg_y}{dt} = \bar{\beta}_y \quad (17)$$

with e.g. $y = t$ for the top quark Yukawa coupling g_t for the top, (for more than one loop we must include the anomalous dimension γ and use

$$\bar{\beta}_x = \frac{\beta_x}{1 - \gamma} \quad (18)$$

and to one loop just the equation

$$\bar{\beta}_x = \frac{\beta_x^{(1)}}{(4\pi)^2}. \quad (19)$$

Then we see that for the couplings in the weak scale range, where $g_t \approx 1$ and $g_3 \approx 1$ too, while the g_1 and g_2 are smaller it is the two terms $-8g_3^2g_t + \frac{9}{2}g_t^3$ that are important. It is actually so that there is a “quasi fixed point” [15] meaning that the terms in the beta-function roughly cancels and the running coupling almost became fixed. Once the qcd-coupling g_3 however would become big as the QCD-scale is approached the important negative term $-8g_3^2g_t$ in $\beta_t^{(1)}$ will of course take over. Thus the running top Yukawa coupling g_t will grow smaller as energy goes up inside this range, and correspondingly of course it will grow stronger as the energy scale is lowered.

Our major point now is that if the energy scale $\mu = \langle \phi_h \rangle$ as given by the Higgs field expectation value $\langle \phi_h \rangle$ is put very low, there will be effectively a very strong running Yukawa coupling $g_t(\mu) = g_t(\langle \phi_h \rangle)$ for the virtual top-quarks that will be present in a vacuum state with such a very small energy scale. This means that their Yukawa fields around these virtual top-quarks will have a strength depending on the running Yukawa coupling and finally the Yukawa field contribution from the mentioned virtual tops in the vacuum will increase as the expectation value of the Higgs field is lowered. It can even at some point increase so fast under the scale lowering that it might *overcompensate* for the decrease of the Higgs field square expectation value $\langle |\phi_h|^2 \rangle$ due to the decrease of the expectation value $\langle \phi_h \rangle$ itself. To decrease this Higgs field expectation value $\langle \phi_h \rangle$ further after that point would no longer pay w.r.t. diminishing the square $\langle |\phi_h|^2 \rangle$ or roughly equivalently the imaginary part of the action S_I . Thus there will be a *non-zero value of the Higgs field expectation value $\langle \phi_h \rangle$, which provide the minimal value for the square $\langle |\phi_h|^2 \rangle$, and thus becomes the favourite value according to our minimizing the imaginary part of the action S_I principle.* This means that our complex action model predicts a small but non-zero Higgs expectation value. In fact this small favourite value of the expectation value $\langle \phi_h \rangle$ is characterized by being the one at which the changes with further change in the scale $\langle \phi_h \rangle$ due to the Yukawa fields around the virtual top quarks in the vacuum and those due to the direct change in the square $\langle |\phi_h|^2 \rangle$ due to the change in the average $\langle \phi_h \rangle$ just compensate.

5.3 Crude estimation of the virtual top quark contribution to the Higgs field fluctuations

Let us first have in mind that since we are interested in a *small/infinitesimal change* in the scale given by the average of the Higgs field it is only the field fluctuations, which

are changed when we change the scale i.e. change the expectation value $\langle \phi_h \rangle$, which matters. Thus we shall most crudely 5e.g. only consider only the virtual top quarks having momenta in a range of this order in order to look for the balance point (where $\langle |\phi_h(x)|^2 \rangle$ has minimum). To be definite we could thus consider instead of all top quarks in the Dirac sea only those with momenta in say a range in which the numerical value of the momentum varies by a factor $e = 2.71\dots$ around a midle value taken as the value $\langle \phi_h \rangle$. Such a range corresponds to a volume in three-momentum-space $\mu^3 * 4\pi$ because the surface of a sphere in three-space is 4π times the radius μ squared. Now we know that in phase space with three degrees of freedom the density of quantum states is $1/h^3 = 1/(2\pi)^3$ (where we have put $\hbar = 1$ so that Plancks constant without the bar is $h = 2\pi$). So the density of Dirac sea Fermions in the momentum range described per unit space volume is $\mu^3 * 4\pi / (2\pi)^3 = \frac{\mu^3}{2\pi^2}$. We should multiply this density by the number of color and spin states of the type of Fermion, we have in mind, here the top quark. For the top quark this becomes a factor 6. Next we should estimate what fraction of a Fourier resolution of the Yukawa field around a virtual top quark taken here to mean one in the Dirac sea with momentum in the range shall be counted as in our chosen range of wavelengths. Sufficiently accurate for our anyway rather crude estimate we might take the part of the Yukawa potential around the (virtual/ or Dirac sea) top-quark lying inside say a distance \sqrt{e}/μ from the topquark position and outside $\frac{1}{\sqrt{e}\mu}$. The idea by this cutting away of the least steep and most steep parts of the potential is that the not so steep part would contribute wave numbers no longer of order μ but rather smaller than μ and the most steep part would contribute mainly to bigger wave numbers than μ . Let us remember that the top Yukawa field is

$$V_{Yukawa} = \frac{g_t/2}{4\pi * r} \quad (20)$$

and so the integral over the square of it r being $1/(\sqrt{e}\mu)$ to r being \sqrt{e}/μ) becomes

$$\int_{1/(\sqrt{e}\mu)}^{\sqrt{e}/\mu} 4\pi r^2 * V_{Yukawa}^2 dr = (g_t/2)^2 \int_{1/(\sqrt{e}\mu)}^{\sqrt{e}/\mu} \frac{dr}{4\pi} \quad (21)$$

$$= \frac{\sqrt{e} - 1/\sqrt{e}}{4\pi * \mu}. \quad (22)$$

To obtain the average contribution to the square Higgs field from these Yukawa fields around the Dirac sea top quarks - in the energy scale range around μ - we shall then multiply this $(g_t/2)^2 \frac{\sqrt{e}-1/\sqrt{e}}{4\pi*\mu}$ with the density of Dirac sea top quarks $\frac{\mu^3}{2\pi^2}$ and by 6 from the number of internal states. So we end up with the product

$$“\phi_h^2” contribution|_{virtual tops} = (g_t/2)^2 \frac{\sqrt{e} - 1/\sqrt{e}}{4\pi * \mu} * \frac{\mu^3}{2\pi^2} * 6 = (g_t/2)^2 \frac{6\mu^2(\sqrt{e} - 1/\sqrt{e})}{(2\pi)^3}. \quad (23)$$

Now we have to note that the scale μ here should be essentially the top mass and thus we take it to be say

$$\mu \approx g_t < \phi_h > . \quad (24)$$

Inserting this we get our expression for the Dirac sea top quarks in the scale range around μ to the Higgs field square average rewritten to

$$“\phi_h^2” contribution|_{virtual\ tops} = g_t^4 \frac{6(\sqrt{e} - 1/\sqrt{e})}{32\pi^3} * < \phi_h >^2 . \quad (25)$$

This were certainly very crude and one could easily imagine for instance an enhancement effect of the type that the top-quarks might attract each other and thus make their Yukawa fields strengthen each other so as to make the fluctuations in the Higgs field - meaning the contribution to the squared Higgs field - become bigger. Such an effect could make the value of the Yukawa coupling needed at the weak scale for the balancing weaker. According to Colin Froggatt and myself [26][27] [28] there could even be a phase transition for some value of the top-Yukawa-coupling g_t so that a new phase of the vacuum, one with a condensate of a bound state of six top plus six antitop quarks, would set in once the coupling g_t at the weak scale were below a certain limit which we estimated to be a critical value $g_{t\ critical} = 1.02 \pm 14\%$. If there is indeed such a phase transition then very likely the minimum in the average of the squared Higgs field $< |\phi_h|^2 >$ as a function of the unsquared average $< \phi_h >$ would occur just at the phase transition value of the top-Yukawa coupling.

5.4 Discussion using a bound state condensate

In any case it is from dimensional arguments relatively easy to see that the minimum in the average of the squared Higgs field as a function of the Higgs field average has to occur for some value of the running top-Yukawa coupling *which is of order unity*, only deviating by various factors π etc. so that we can take our model to be at first successful just from the mere fact that the experimental top Yukawa coupling at the weak scale is indeed as needed of order unity. In fact it is $g_t = 0.935$. If indeed the experimental coupling by nearer calculation should turn out to be as Froggatt and I claim, but disputed by the Stony Brook group [29], the phase transition coupling it would be a real support for the idea of the present article. It would namely as already just said very likely be the minimum in the squared Higgs field average just at the phase transition value. In that sense the minimization of the squared Higgs field average could automatically directly lead to the existence of two degenerate phases in this special case of the phases with and without the 6 top plus 6 antitop bound state condensate. But this would not be so new in the sense that Ninomiya and I already published an article [2] suggesting that in general the model minimizing the imaginary part of the action S_I gets allowed to even tune in some coupling constants would almost unavoidably lead to the “multiple point principle” [30]. So the case of the degeneracy of the bound state condensate vacuum and the one without - in which we presumably live - would only be a special case of that. The present article argument for solving in a way the hierarchy problem by giving a mechanism leading to the finetuning of the Higgs mass scale to be exponentially small is also then not quite new in as far as we in our earlier work[28] used the multiple point principle as a supposed principle to argue for the top-Yukawa coupling to have to have the critical value at the weak scale, and used that to fix this weak scale. Rather would should consider it that I in the present article bring the same numerical coincidence - namely that the Yukawa

coupling for the top-quark at the weak scale is just the critical one causing the phase transition - in to confirm the complex action model in a slightly different and one could say more direct way.

5.5 Discussion using the crude calculation

Although it is very likely that some phase transition could be involved in making there be a minimum in the average of the squared Higgs field $\langle |\phi_h(x)|^2 \rangle$ for $\langle \phi_h(x) \rangle \neq 0$ we could also just continue our estimate using (25). We would have to compare (25) with the corresponding contribution to the average squared field $\langle |\phi_h(x)|^2 \rangle$ from the average simply coming in via $\langle \phi_h(x) \rangle^2$. Let us take from the range considered this contribution as

$$\phi_h^2 \text{contribution}|_{naive\ classical} = \langle \phi_h(x) \rangle^2 (e - 1/e). \quad (26)$$

Then the balance condition which should be satisfied in order for the field average $\langle \phi_h(x) \rangle$ to be at the minimum we discuss would be

$$\begin{aligned} \text{"}\phi_h^2\text{" contribution}|_{virtual\ tops} &= g_t^4 \frac{6(\sqrt{e} - 1/\sqrt{e})}{32\pi^3} * \langle \phi_h(x) \rangle^2 \\ &= \langle \phi_h(x) \rangle^2 (e - 1/e) = \phi_h^2 \text{contribution}|_{naive\ classical} \end{aligned} \quad (27)$$

This condition now is easily transformed into the condition

$$g_t^4 * 6 = 32\pi^3(\sqrt{e} + 1/\sqrt{e}) \quad (29)$$

From here we then get

$$g_t = (165.4(1.649 + 0.6065))^{1/4} = 373.0^{1/4} = 4.39 \quad (30)$$

This our predicted g_t -value is a bit bigger than the experimental value $g_t|_{exp} = .935$, but taking the crudeness into account it is not so bad. If indeed as suspected from the bound state speculations there were an effect of the virtual top-quarks clustering together the Higgs field surrounding a cluster of virtual tops would get its *square* of its strength - as we used above - increased by a factor being the effective number of virtual tops in the cluster. This factor would in our estimate come to decrease our value for the fourth power of the g_t by the number of virtual tops in the cluster. If we for example optimistically could have the effective number of tops in the cluster being the 12 that were the number of constituents in the bound states of 6 top + 6 anti tops we would decrease our prediction for g_t by the fourth root of this 12. That would bring the prediction down from our first 4.39 to 2.35. It is still not perfect but really we cannot expect better accuracy with the just made estimate.

In any case it is now surely a possibility that it were somehow arranged that the imaginary part of the action were minimized as far as the vacuum Higgs field square is concerned. This clearly supports our complex action model.

6 Worrying about too much arrangement of the state of the universe

There is a severe problem with our complex action model: If really the integral $\int |\phi_h|^2 d^4x$ of the Higgs field square integrated over all space time should be so important in fixing the history of the universe that even a moderate number of Higgses that would have been produced in the SSC[31] machine should have effectively influenced the political history of this machine, then the much more common particles that are surrounded by Higgs fields like the quarks and the leptons would make much more dramatic contributions a priori. It thus seems at first that depending on how the sign of the relative extra Higgs square contributions caused by such more common reactions transforming particles, that can change the Higgs square field around them, the total set of particles involved would be pushed completely to one side in order to minimize the integrated Higgs field squared. Let us to be concrete look at neutron decay. It is the reaction

$$n \rightarrow p + e + \bar{\nu}_e. \quad (31)$$

It is now not really so difficult to calculate at least crudely the difference in the total change in the Higgs field square integrated over space time - or better just over space - when the above process takes place. Depending on the sign of the shift in the Higgs field square caused by the process(31) it should after our complex action theory be arranged if at all possible - by adjusting the initial condition and the coupling constants allowing adjustment - to be so that the process were driven (almost) totally to one side. If for instance it turned out that the neutron by its interaction with the Higgs field made the Higgs field squared diminish more around the neutron than the corresponding diminishing around all of its three decay products added up, then there should be no decay product combinations left and somehow all should have been arranged now to be in the form of neutrons. This would be very difficult indeed to arrange by appropriate initial conditions, since how could even a “God” able to arrange initial conditions keep neutrons stabilized against decay if there were no protons left to keep them bound into nuclei? If on the other hand the difference should have been so that the decay products of the neutron would be more favorable by suppressing the Higgs field square than the neutron, then there should have been arranged to be no neutrons left. That would mean it should all be hydrogen, also not so easy to keep, for would there not develop stars? And the stars would presumably make helium etc. with lots of neutrons inside.

None of the two scenarios are very easy to realize, and certainly none of them are true experimentally! At first it seems that we here have a severe problem for the complex action theory! There is, however, one way out: If it had been arranged that the coupling constants were just tuned in to be so that the amount of suppression of the Higgs field square were just the same by the set of all the three decay products and by the neutron itself, then there would be no need for bringing the process to one side or the other. It were in fact the main point of my paper connected with my talk in Spaatind [13] that one could expect that our complex action theory could easily be expected to precisely arrange such an equality of the Higgs field square suppression from the particles on the two sides of a transition equation, such as (31). Indeed I claimed to find approximate

experimental evidence for the relations between quark masses proton mass etc. being deduced from requiring this relation to be true. That is to say I claimed that it had somewhat miraculously, one could almost say, been arranged that whether the neutron decays or not, will make no change in the Higgs fields square integral over space. By this almost miraculous adjustment the need for having the reaction pushed almost totally to one side in our model gets dispensed of. This balance making the neutron decay irrelevant for the Higgs field square integral is what rescues our model for the problem with this reaction, neutron decay. But there are other reactions that could threaten our model by having to be pushed quite to one side as far as possible or by having associated with them another fine tuning of couplings arranging, that the reaction in question just does not cause change in the Higgs field square suppression. A number of relations of this type might be found, and it would of course be very interesting, if they are fulfilled in nature.

7 The relation

So far I only looked properly on the relation expected to avoid that the neutron decay reaction should be pushed completely to one side. To write that needed relation I did estimate in the article [13] the amount of change in the Higgs field square around the neutron and on the other hand the sum of the changes around the three decay products, the proton, the electron and the neutrino. The relation we should derive is then the relation needed to keep the integrated square of the Higgs field the same before and after the neutron decay, so that letting the neutrons decay does not matter for the value of S_I which is the space time integral of the Higgs field square.

7.1 For the Relation Computational needed Assumptions

Let me here make a few remarks about this calculation:

- 1) We know that the Higgs Yukawa field around a quark or a lepton is proportional to the Yukawa coupling for that quark or lepton, or W etc.
- 2) and we suppose that we can Taylor expand so as to argue for that the change in this square of the Higgs field is indeed also proportional to the Yukawa coupling. This is actually not so obvious for two reasons, but we still shall use this approximation:
 - A) We strictly speaking use that the change in the Higgs field around the quark say is small compared to the Higgs VEV(=the vacuum expectation value). This is o.k. for the light quarks and the electron in the dominant region in their Yukawa potentials.
 - B) The idea also mentioned in this article that the Higgs vacuum expectation value should precisely be the one leading to a minimum in the Higgs field square, strictly speaking tells that precisely that derivative (of the square w.r.t. the field itself), we assume to dominate the Taylor expansion in calculating the effect of the Yukawa field around a quark say, is zero. There may though be some excuses for not taken that too seriously:

- * B1) The Yukawa potential is not constant in space so that spatial Fourier components relevant are not exactly that $\vec{p} = 0$ component, for which the derivative is zero.
- * B2) As we speculate there could likely be an, although weak, first order phase transition just at the minimum for $< |\phi_h|^2 >$ so that there is at least slightly non-zero derivatives from right and from left, but no true zero-slope place. Then the sign of the derivative would depend on from which of the two meeting phases we work. The speculation is that the phase relevant for the Higgs field average below the minimum point is a phase with a condensate of bound states, while we above have the “normal” phase without such a condensate. It should then be the right derivative, meaning the one in the “normal” phase that we shall use in the Taylor expansion.
- * B3) One could also expect an indirect effect of the Higgs field square coming as a loop correction via an effective complex kinematical term coefficient.
- C) So from A) and B) the change in the Higgs field square integrated over 3-space is proportional to the Yukawa coupling of the particle that makes the change.
- D) The integral over space of the Higgs field (squared or not) is in addition proportional to the inverse of the γ for the particle. We could say it simply reflects the Lorentz contraction, correcting the volume crudely covered by the Yukawa potential by the factor γ^{-1} .
- E) The appearance of the just mentioned Lorentz contraction factor means that the neutrino, which in practice always runs with almost exactly the speed of light (even if there are extremely tiny neutrino masses different from 0.) gets totally Lorentz contracted, and so the neutrino can be left out, so that we only have to consider it as if the neutron decayed into an electron and a proton alone.
- F) Thinking as is typically what happens that we have to do with non-relativistic neutrons, protons and electrons in the bulk of the universe, we have no Lorentz contraction factor for the electron, but even though the proton and the neutron are essentially at rest the quarks inside them move with speeds comparable to that of light.
- G) It is indeed the major calculational trouble of making our relation to estimate the appropriate average of the γ^{-1} factors for the quarks inside the proton and the neutron.
 - * G1) In crudest approximation for the average of the correction factor for Lorentz contraction of the Yukawa-field around a quark is $< \gamma^{-1} > \approx < \gamma >^{-1} = < E_q/m_q >^{-1}$ where E_q is the energy of the quark inside the, say, proton and m_q its (current algebra) mass.
 - * G2) In the article [13] I gave a name

$$“ln” = < \gamma > < \gamma^{-1} > \quad (32)$$

to the correction to this just mentioned approximation, and in this article are mentioned estimates like " $ln'' = \frac{\ln 2 <\gamma>}{2 - <\gamma> - 1}$ ", which for $<\gamma> = 55$ would give " $ln'' = 2.37$ " while for $<\gamma> = 27.5$ it gives " $ln'' = 2.05$ ".

- * G3) Another difficulty that needs further estimation is to estimate how big a fraction of say the proton mass is actually sitting as energy of the valence quarks, so that we can use one third of the amount as the average $<E_q>$ of the quark energy E_q in the proton rest frame. It is presumably not quite unreasonable to estimate that about *half the proton mass* sits as energy on the three valence quarks together, so that we can take (crudely of course) $<E_q> \approx m_{proton}/(2 * 3) \approx 160 MeV$.
- * G4)(Current algebra) quark masses are of course not extremely well determined for the light quarks u and d , with which we are here concerned, but we may take:

$$m_u = 1.7 \text{ to } 3.3 MeV \quad (33)$$

$$m_d = 4.1 \text{ to } 5.8 MeV \quad (34)$$

7.2 Agreement of our relation involving quark masses etc

From this procedure I then arrived in article [13] to the relation

$$\sqrt{m_d^2 - m_u^2} = \sqrt{E_q m_e} / "ln'' \quad (35)$$

which is relatively well satisfied, if we take the quark masses (34), $E_q \approx 160 MeV$ and " $ln'' = 2.37$ ". In fact then we would get (using $m_e = 0.511 MeV$)

$$R.H.S. = \sqrt{E_q m_e} / "ln'' = 3.81 MeV \quad (36)$$

$$L.H.S. = \sqrt{m_d^2 - m_u^2} = \sqrt{13.9 \text{ to } 22.75} MeV \quad (37)$$

$$= 3.7_3 \text{ to } 4.7_7 MeV. \quad (38)$$

Really it should be possible to extract a better estimate for the $<\gamma^{-1}>$ and $<\gamma>$ from using the parton distribution function(PDF) for the nucleons. Actually looking at the PDF's for the light valence quarks one sees a peak at the Bjorken[16] momentum fraction $x = 0.2_0$ [17]. That should be taken to mean that the typical energy of a valence quark, u or d , should be $E_q = 0.2_0 m_N = 0.19 GeV = 190 MeV$. Inserting this E_q -value instead of the 160 Mev would bring the right hand side of our relation up to

$$R.H.S. = \sqrt{E_q m_e} / "ln'' = 4.15 MeV. \quad (39)$$

This is very much inside the range of uncertainty of the quark masses, so our agreement is very good!

If the (current algebra) quark masses were known better it would be worthwhile to evaluate say $<\gamma^{-1}>$ more accurately from using PDF's and/or other informations so as to test our relation more accurately.

8 A Few Further Evidences for the Minimization of the Integral of the Higgs Field Squared

8.1 Why Nuclear Binding Ends up Small Compared to the Kinetic and Potential Energies Separately

Similarly to the above discussed worry, that in our model there should either have been no neutrons or no possibilities for combining a proton, an electron, and an antielectron neutrino, we shall also ask, if there ought to have been either only nucleons bound into heavier nuclei or only unbound nuclei meaning only hydrogen. The point is that, if the suppression of the Higgs field square by a bound and an unbound nucleon is not the same, then the principle of minimizing the square of the Higgs field integrated over space and time would either lead to an arrangement, so that there were only bound nucleons or to one, in which there were only unbound nucleons. Providing our already derived relation (7) it should not matter for this purpose whether the nucleon is a proton or a neutron (corrected for the accompanying electron for charge neutrality the proton (with its electron) and the neutron would namely suppress the Higgs field equally much).

Our complex action model would now have a problem unless it happens by some finetuning or for whatever - “arranged” - reason that the Higgs field square decrease due to the presence of a nucleus is the same integrated over space as if we have the same nucleons separated out as free nucleons. Thus we must now at least very crudely estimate how it should be in order that these two suppression amounts should indeed become the same number for $\int |\phi_h|^2 d^3\vec{x}$, i.e. this quantity would be changed equally much by the nucleus as by its constituents being free (and with low, nonrelativistic, speeds).

To make a first very crude estimate of the difference between the two suppression amounts we remark, that assuming the quantity (32) not to vary significantly because of the binding taking place the fulfilment of the requirement amounts to that the energy carried by the quarks (kinetic and mass energy of the quarks) should be the same whether the nucleons are bound or not. In the approximation that the energy of the non-quark energy nature (meaning say the gluon carried energy) being constant or proportional to that for the quarks we would then say that in the very crudest approximation the energy change by the baryons being bound into heavier nuclei should zero, i.e. small. The degree of smallness is of course given by the deviation from correctness of the assumptions we made here. Let us take the smallness to mean though, that the binding energy that should be small should be that compared to the kinetic energy or the potential energy separately. Interpreted this way that is in fact what somewhat surprisingly is wellknown to be the case for nuclear forces. In spite of the Fermi kinetic energy of the the nucleons in the nuclei and the potential energy being both rather large, the Fermi kinetic energy being 38 MeV per nucleon [18] typically the NET binding energy per nucleon is ONLY in the most stringly bound region of nucleides 8.8 MeV[19]. This means the net binding is down by a factor $38/8.8 = 4.3$. That is to say that in the first approximation there is indeed very crudely an approximate cancellation so that if as we assumed crudely the energy of the quarks follow that of the nucleons (proportionally) in such a way that even the potential energy of the nucleons gives rise also to kinetic and mass energy of the quarks, then the

quark energy would not change by the binding. But that in turn were what we needed to not get contradictionally to truth that either all nucleons should be bound or oppositely none being bound.

It would be extremely interesting for checking our model to estimate more accurately how the gluonic part of the energy gets changed by binding the nucleons into nuclei and how our crude estimate should in addition be corrected. Then one could namely hope to calculate what precise value of the binding energy (compared to say the kinetic energy) would be required for our model to be consistent. It would of course be an interesting victory, if such a calculation would lead to the experimental binding energy of the nuclei.

8.2 Could we even Predict the Outcome of the Debate on Nuclear Power?

One would expect that the nuclear power production processes a priori would change the bindings of the nuclei in detail. So although the binding of nuclei to very first approximation may well have been tuned in to make no change in the integrated Higgs field squared under such processes, it would presumably require an extra little fine tuning to get the Higgs field square integrated over space totally independent of whether nuclear power is being used or not. If we can calculate the difference in change of the Higgs field square integrated over space for the nuclear power fuel relative to the nuclear power waste, we should be able to estimate how the squared Higgs field would be arranged to be the smallest integrated over space time, by having nuclear power being used or not. If we for example find that the waste had the smallest Higgs field square integral, then our model would predict that by “almost miraculous accidents” the political development would be so as to make nuclear power come to be used. If we get the opposite result that it is fuel that gives the lowest integral of the square of the Higgs field, then the use of nuclear power should be “almost miraculously by accident” stopped.

8.3 Higgses Produced in Cosmic Ray Impacts

In the earlier article [13] I argued for that using e.g. theoretical estimates of Tully [24] and extrapolating them linearly as the cross section being a linear function of the center of mass energy \sqrt{s} of the colliding nucleons one could define crudely “an effective Higgs production threshold” as that value for the center of mass energy \sqrt{s} for which the extrapolated cross section passes zero. Extrapolating this “effective threshold for Higgs production” as a function of the Higgs mass to the range favoured by our own prediction [25] of the Higgs mass $M_h = 120 \text{ GeV}$ an effective threshold of $\sqrt{s}|_{\text{threshold}} = 2.7 \text{ TeV}$ is reached. That in turn when translated to scattering on a fixed target as a proton coming in and hitting the atmosphere gives a Higgs production effective threshold around $3.6 * 10^{15} \text{ eV}$ and this is very close indeed to the “knee” in the Gaisser curve of the cosmic ray intensity as a function of the energy at about $2 * 10^{15} \text{ eV}$. That means that the fall off with extra speed of the cosmic ray intensity signaled by this “knee” is as if it were just “chosen” to avoid producing Higgses effectively! It is even so that especially just above the “knee” the cosmic radiation seems more rich in presumably iron ions than the lighter elements supposedly dominating below the “knee”. For iron of course the “effective Higgs threshold” would be

higher in energy by a factor of the order of the ratio of the iron to light nuclei atomic weight. at extremely high cosmic ray energies there is even the “angel” (the spectrum flattens off again) so there are some although remarkably few cosmic rays producing Higgses. Thus our model does NOT fully prevent Higgs production in cosmic ray, if it were true.

The coincidence order of magnitudewise of the “knee” with the “effective Higgs production threshold” (for an almost by now unavoidable Higgs mass range) would be yet an interesting coincidence supporting the principle of the Higgs field squared being kept minimal, since of course the precense of genuine Higgs particles means an increase in the Higgs field square.

Really of course it would have been better in supporting the complex action model to have no cosmic ray above the knee at all, but we only got the “knee” and then there even were the “angel”, which however very likely consists of cosmic rays from further away galaxies, while the cosmic ray below the “knee” is likely from our own galaxy. So the fraction of the very high energy cosmic ray particles actually hitting any atmosphere or any astronomical object would be lower than for the particles below the “knee’.

9 Conclusion of much evidence for the complex action indirectly!

The new result of the present article is the claim that it is possible that the surprisingly little but not - even compared to Planck scale - totally zero expectation value of the Higgs field $\langle \phi_h \rangle = 246 \text{ GeV}$ is indeed that value of this expectation value which leads to the minimal value of the related average $\langle \phi_h^2 \rangle$ of the *square* of this same Higgs field. We imagine indeed that there is some parameter of the vacuum or say some coupling constant that can at least be imagined to be variable somehow and which thereby leads to a correlated variation of both the average and the square of the average of the Higgs field. I.e. we shall think of both $\langle \phi_h \rangle$ and $\langle \phi_h^2 \rangle$ as depending on the same such parameter. Then our claim that basically inside the Standard Model - except for this “variability” - the parameter has that value which makes the average of the *square* the smallest possible! Indeed we claimed that this minimal average of the square is achieved - since its for us most interesting dependence comes from fluctuations due to virtually present top-quarks - when the running Yukawa coupling for the top-quark has a certain specific value, which is of order unity, but which we could not so far calculate so accurately as would have been useful. It is nevertheless if one accepts some physical model/story behind a solution to the problem of the large scale ratio of the weak scale to say the Planck scale. That is a solution very similar to the one which C. D. Froggatt, L. V. Laperashvili and earlier presented [28]. The crux of the matter is that by relating the weak scale to a *running* coupling - in the present work the top-Yukawa coupling - it gets naturally of an exponential size, because the running of couplings are “logarithmic”, logarithmically slow.

The story behind the principle of minimizing the average of the square of the Higgs field, which we used, is the “complex action model” of Ninomiya and myself. In this model we namely arrive to minimizing the imaginary part of the action $S_I[\text{history}]$ calculated

for the actual history *history* of the Universe. We then make the approximation that the vacuum dominates and that the mass square term in the imaginary part of the action S_I dominates. Thus the average square of the Higgs field multiplied by the space time volume becomes approximately the S_I and thus minimized when the average of squared field is minimal. Thus the average of the (unsquared) Higgs field should get adjusted to minimize the average of the squared field.

We reviewed an earlier work of mine also made on the hypothesis that the average of the square of the Higgs field should be minimized:

If say the neutron versus the decay product of a neutron made different change in this squared Higgs field integrated over all space and time, then the most beneficial in the sense of the smallest Higgs field squared integrated, would either be a history with no neutrons at all, or one with no decay products of neutrons meaning no electrons and protons. But in Nature we do find that the reaction of neutron decay is in this way pushed to one side or the other. Rather there are Nature, both combinations of electron and protons and even antineutrinos, *and* neutrons! The reaction is *not* pushed to one side. According to our minimization of S_I principle this reaction not being pushed to a side is *only possible provided the modification of the Higgs field (square) by a neutron and by its decay products is the same* !

Then I rewrote a formula needed for this modification of the Higgs field squared and integrated be the same for the neutron and for its decay products, and lo and behold this relation is fulfilled inside our estimation accuracy!

We should consider these two numerical agreements of the principle of minimizing the integral over space and time of the Higgs field square as a success for the hypothesis that such a quantity were indeed *minimized* !

The two cases thus argue for that some parameters of nature are indeed adjusted towards a minimization of some quantity like the integral over space time of the Higgs field squared and integrated.

If we include the more weak agreements we have further somewhat supporting successes mentioned above:

1) The smallness of the binding of nuclei compared to say the kinetic energy of the nucleons inside the nuclei could be adjusted so as to make the change in the over space integrated Higgs field (squared) be independent to whether the nucleons are bound or not. We can consider the fact that the binding energy of nuclei is relatively small compared to the Fermi-energy and the potential energies separately as suggestively making it more likely that the suppression of the Higgs field squared could have been finetuned to be the same whether the nucleon is bound or not.

2) The “knee” in the cosmic ray intensity curve versus energy is “made” to cut away Higgs production in the atmosphere of astronomical objects being hit by the cosmic radiation.

3) The bad luck of the S.S.C. machine[31] that would have produced a lot of Higgses, but which were killed by the Congress of the United States.

If we further take it that an imaginary action theory could be used to argue for the Multiple Point Principle (MPP)[30], then successes of this principle, that there should be many degenerate vacua - or rather one vacuum should be just about to decay into another one (called Meta-MPP[?]) -, could be counted as successes of the complex action

model too. Now the claims of success of such a principle of essentially degenerate vacua counts a Higgs mass as low as allowed in the Standard Model, a prediction seemingly getting slowly more and more likely as higher masses for the Higgs get excluded. We have also claimed that it could give the top-Yukawa-coupling if we believe in the possibility of forming certain bound states of six top and six antitop quarks. We even claimed already that this MPP could “solve the problem of why the weak scale is exponentially small compared to the say Planck scale”. The main point of the present article is really a slightly different version of the same prediction - an exponentially small weak scale - but in a somewhat different setting in details, although both ways of solving this scale problem comes directly (this article) or indirectly (our earlier articles with Laperashvili and Froggatt[28]) from the complex action model.

Taking just some of these “evidences” for the complex action [1][2] seriously would mean that there *is* some remarkable evidence for the truth of such a model or some similar model with similar predictions, especially a minimization of the Higgs field squared would be called for.

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